

# About the Textbook

This text is organized around numbers and arithmetic. The topics are covered roughly in the order they are developed in elementary school. The first three chapters cover most of the arithmetic of Grades K-3. Chapters 4 and 5 jump ahead to topics (prealgebra and prime numbers) usually covered in Grades 6 and 7; that jump allows us to review ideas about algebra and proofs which are needed for a “teacher’s understanding” of the subsequent material. Chapters 6 and 7 return to the original timeline, developing fractions (as is done beginning in Grades 3 or 4) and the follow-up topics of ratios, proportions, percentages, and rates. The last two chapters complete the development of elementary arithmetic by discussing negative and real numbers.

The textbook is divided into short sections, each on a single topic, and each followed by a homework set focused on that topic. The homework sets were designed with the intention that all or most of the exercises will be assigned. Many of the homework exercises involve solving problems in actual elementary school textbooks (the ‘Primary Mathematics’ books described below). Others involve “studying the textbook” — carefully reading a section of the book and answering questions about the mathematics being presented, with attention to the prerequisites, the ordering, and the variety of problems on that topic. Both types of exercises will help you develop a teacher’s understanding of elementary mathematics.

## Supplementary Texts

This textbook is designed to be used in conjunction with the following five Primary Mathematics books (all are U.S. Edition).

- Primary Mathematics 3A Textbook (ISBN 981-01-8502-2)
- Primary Mathematics 4A Textbook (ISBN 981-01-8506-5)
- Primary Mathematics 5A Textbook (ISBN 981-01-8510-3)
- Primary Mathematics 5A Workbook (ISBN 981-01-8512-X)
- Primary Mathematics 6A Textbook (ISBN 981-01-8514-6)

These books were developed by the Curriculum Planning and Development Division of Singapore’s Ministry of Education, and published by Federal Publications. While these books were initially created for Singapore elementary students, they have been adapted for use in other countries, including the United States. We will refer to them as “Primary Math 3A”, “Workbook 5A”, and so on. They can be purchased from the distributor in the U.S. at [www.singaporemath.com](http://www.singaporemath.com).

The Primary Mathematics series is printed as one course book per semester, each with an accompanying workbooks. The semesters are labelled ‘A’ and ‘B’ , so ‘5A’ refers to the first

semester of Grade 5. In each grade, the first semester focuses mainly on numbers and arithmetic, while the second semester focuses more on measurement and geometry.

## Why the Primary Mathematics books?

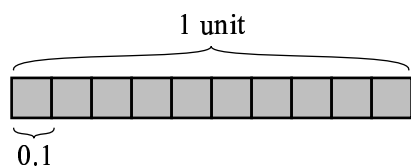
The aim of this course is to develop an understanding of elementary mathematics *at the level needed for teaching*. The best way to do that is to study actual elementary school textbooks and to do many, many actual elementary school mathematics problems. The Primary Mathematics books were chosen for that purpose.

We will read and study these books with two goals in mind: understanding the mathematics and understanding the curriculum development. The Primary Mathematics books give an extraordinarily clear presentation of what elementary mathematics is and how it is organized and developed. They lay out the subject in depth, and they include a rich supply of exercises and word problems. The mathematics is always clean and correct, and topics are repeatedly covered from different approaches. Viewed from a broader perspective, these books provide much useful guidance about curriculum issues. They exhibit the principles of a well-designed curriculum better, it seems, than any textbook series currently available in English.

It is not surprising, then, that the Primary Mathematics books are also successful with children! The Third International Mathematics and Science Study (TIMSS) rated Singapore's elementary students the best in the world in mathematics (it also found that the curriculum is highly coherent). These beautifully designed books are a major factor in student success.

As you read and do problems from these books, notice the following:

1. The absence of clutter and distraction. These books contain mathematics and nothing but mathematics. The presentation is very clean and clear, and is done using simple, concise explanations.
2. The coherent development. Each topic is introduced by a very simple example. It is then incrementally developed until, quite soon, difficult problems are being done. Topics are revisited for 'review' and the level of the mathematics is constantly ratcheted upward.
3. The short, precise definitions. The children pictured in the margins give the precise definitions and key ideas in very few words. These 'student helpers' often clearly convey an idea that might otherwise take an entire paragraph!



Divide 1 unit into 10 equal parts.

Each part is  $\frac{1}{10}$  or 0.1.

4. The "concrete  $\Rightarrow$  pictorial  $\Rightarrow$  abstract" approach. This approach results in a very clear introduction to a topic.
5. The books serve as teacher guides. The books make the mathematical content of each lesson clear to the teacher and help teachers plan lessons. They also provide examples and activities to be done in class and allow teachers flexibility in designing lessons.

That first point above should be stressed. The Primary Mathematics books are deliberately focused. They contain no distracting extras such as long introductions and summaries, biographical stories, explorations, or discussions of non-mathematical topics. Homework is relegated to workbooks, and group projects and explorations are put in separate teacher guides. The pictures effectively convey meaning; they are not there for stylistic reasons. The judicious use of white space makes the books easy and enjoyable to read. The resulting short textbooks keep young students focused on learning mathematics.

If you compare the Primary Mathematics books to other elementary textbooks, you will appreciate these points. Many textbooks feature distracting side-bar messages, unnecessary drawings, showy photographs and highlighted boxes, and frequent font changes. The mathematics is obscured and perhaps lost altogether.

Study and enjoy these books — and keep them! When you become a teacher, these books will be a valuable resource, helping with explanations, providing extra problems, and giving guidance in how to present mathematics.

## Reading this Textbook

Students reading the Primary Mathematics books interact with the books at the places indicated by colored boxes. As explained in the preface of each book, the colored ‘patches’ are prompts for student participation and class discussions. They occur in relatively easy exercises, where they encourage active learning and allow students to check their understanding.

This textbook also includes “learning exercises” exercises embedded in the text, many with boxes  $\square$  of various sizes prompting you to answer. Some of these exercises will be discussed in class, but usually you will encounter them while reading on your own. Do these exercises as you read! Most only take a minute or two. Pencil your answers next to the boxes (the boxes themselves are usually not large enough to hold answers). These exercises are designed to clarify the text and help make mathematical discussions more concrete. Some mathematical ideas are difficult to communicate in words, but quickly become clear by *doing problems*.

That same principle — that mathematics is best learned by solving problems — applies to the course as a whole. Read each section of the textbook, but leave plenty of time for doing the homework sets. They are the most important part of the course.

## The Homework Sets

This course is built around homework problems from the Primary Mathematics books. As you do homework, bear in mind that the goal is not merely for you to *do* the problems, many of which are not hard. Instead, the goal is to think about problems from the perspective of a teacher. Teachers must be able to identify the key steps in solving a problem, so they can guide and prompt students. They must also be able to give clear, grade-appropriate presentations of solutions. Try to bring out these teaching aspects of problems in your homework solutions. In general,

Make your answers clear, concise, legible, and simple. They should look like an answer key to be handed out to an elementary school class.

This idea — clear, concise solutions — is one of the main themes of this course. You will learn many tricks and teaching devices which will help you craft such solutions, including models (introduced in §1.1), number bonds (§1.4), bar diagrams (§2.2), and “Teacher’s Solutions” (§2.3). These devices convey mathematical ideas without words, making short explanations possible. In mathematics, longer explanations are often *more confusing*. Consequently, you should avoid writing out paragraph-long explanations — short solutions are less work for you and are clearer to students (and to your instructor!) If you practice brevity in your homework solutions you will find yourself becoming increasingly comfortable giving teacher-quality mathematical explanations.

## Mathematics not covered

This course focuses on arithmetic. Elementary mathematics also includes a second main set of topics centering on measurement and geometry. Those topics, listed below, will be covered in another course.

Measurement and the metric system.  
Introduction to lines and angles.  
Pythagorean theorem.  
Congruence and similarity.

Area and perimeter.  
Volume, surface area, and density.  
Basic probability and statistics.  
Linear and quadratic equations.

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