Elementary Mathematics for Teachers, 2015 reprinting.
by Parker and Baldridge.

Answers to Selected Problems

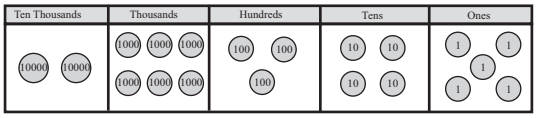
Below are answers, solutions, hints and strategies for solving some of the homework problems.

Homework Set 1

1. (a) 38, 3213, ... (c) 3733, 1970, 2059, ... (d) LXXXVIII, CXLIX or CXXXXVIIII, ...
2. 17 fewer symbols.
3. (d) Replace the 12 tallies with ∩||.
5. (a) 6,000,003,408.
   (c) 23326.
7. Method: replace each | by a ∩, each ∩ by a ᘐ, and each ᘐ by a ᘓ.

Homework Set 2

1. a) Expanded.
   b) Hundreds.
   c) The tens digit. So that the only difference is place value.
   d) (7a) Hundreds, Thousands. (7b) Tens, Thousands.
   e) 0.
   f) (9) 5073, 4973; (11) 4123, 3412, 3142, 2431;
2. a) We must fill in the blanks to maximize
   __ thousands + __ hundreds + __ tens + __ ones.
   Strategy: begin with the thousands place and work to the right, putting the largest available digit in each place value.
   Explanation: putting the 8 in the thousands place is the only way to get a number larger than 8000, etc.
   b) 2078 (write the numbers in order from smallest to largest, avoiding an initial 0.)
   c) 1736, 7504, 90, 800, 3, 900.
   d) (1) less, greater, greater, less, less, less.
   e) subtract 1000; thousands.
3. a) the teacher.
   c) Textbook 4A, p. 10, Problem 7a:



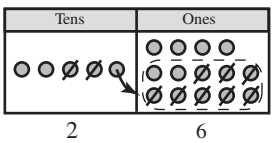
   d) (8a) 5100, 5300, 5700, 5900, 6400.
   e) (10c) $15,000 - 6000 = (15 - 6)$ thousands $= 9000$.
6. a) 89, 245.
   c) $(1101)_5$.

Homework Set 3

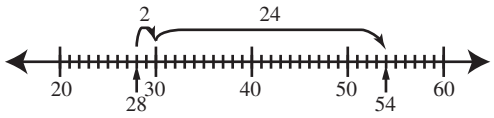
2. a) Compensation: $86 \overset{14}{\curvearrowleft} + 34 = 100 + 20$.
   d) Any-order, *or* associative and commutative
   e) Any-order, *or* commutative.
3. e) $34 + 17 + 6 + 23 = (34 + 6) + (23 + 17) = 40 + 40 = 80$.
4. b) $19 + 21 = 2 \times 20 = 40$.
8. a) "Ryan has \$2" or "Ryan's money: \$2." (Ryan himself is not *equal* to \$2!)
   b) $4.8803 \approx 4.8$

Homework Set 4

2. a) The girl at the bottom of page 18 gives the definition: the difference of two numbers is the larger number minus the smaller number. The difference between 9 and 3 is $9 - 3 = 6$; the difference between 3 and 9 is also $9 - 3 = 6$.
   b) (i) (4) Part-Whole. (6) Comparison. (7) Comparison.
6. a)



b)



54 - 28 = 2 + 24 = 26

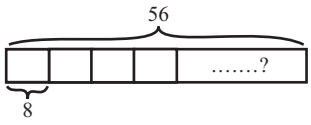
7. a) Jill had cookies. Her brother ate 7 of them. How many does she have now?
8. a) See the timeline on page 14.
   b) The subtraction problems from "0 - 0" to "20 - 10."

Homework Set 5

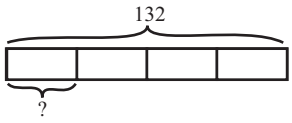
1. a) Commutative.
   b) Set; measurement model; . . . .
2. a) Flash cards. To memorize the 1-digit multiplication facts.
5. $24 \times 15 = 24 \times (10 + 5)$
   $= 240 +$ half of 240
   $= 240 + 120$
   $= 360.$
7. d) For each digit 1, . . . , 9, when you bend the finger in that digits place, the number of fingers to the left is one less than the digit, and the number to the right is the ten's complement of the digit.
8. a) $5 \times 87 \times 2 = 87 \times 10 = 870$

Homework Set 6

1. a) PD. b) MD. c) MD.
3. a)



b)



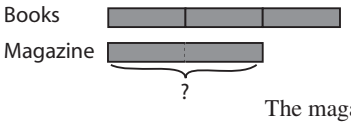
4. a) Your word problem should ask, "84 is how many 21's?"
   b) Your problem should ask, "91 is 5 equal groups of what size?" For example, "5 friends share $91 dollars equally. How much does each get?" Notice that the answer ($18.20) involves decimals or fractions; for this reason partitive division with remainders is not done until decimals or fractions are introduced.

Homework Set 7

1. a) $(26 + 83) + 54 = (26 + 54) + 83 = 80 + 83 = 163.$
2. c). $3200 \times 34 \div 16 = \frac{3200}{16} \times 34 = 200 \times 34 = 6800$
3. b) $62 - 39 = 63 - 40 = 23$
   d) $1500 \div 25 = 6000 \div 100 = 60$
4. c) $7855 - 723 = 7132$
6. b) PV. c) PV or DP. f) X. h) PV.

Homework Set 9

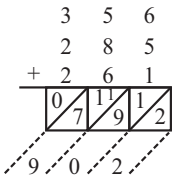
2. Exercise 9.1: Number of white beads: 274-150-70 = 274-220=54. There are 70-54=16 more white beads than red beads.
   Exercise 10.2: There were $45 \div 3 = 15$ groups of 3 cards.
   Total cost: $15 \times \$2 = 30$ dollars.
   Rebundled as $45 \div 5 = 9$ groups. Sold for: $9 \times \$4 = \$36$.
   Peter made a profit of $36 - 30 = 6$ dollars.
3. Exercise 9.3: Problem 8.



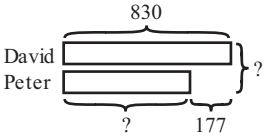
Cost: $30 - 5 = 25$ dollars.
5 units = $25.
1 unit= $5.
The magazine cost 2 units = 10 dollars.

Homework Set 10

1. c)



3. Textbook 3A, page 23, Problem 9:



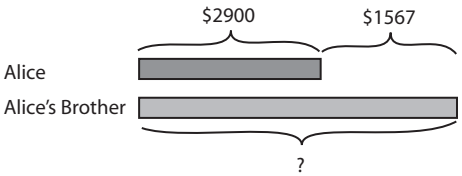
$830 - 177 = 653.$
$830 + 653 = 1483.$

Peter collected 653 stamps.
They collected 1483 stamps altogether.

5. Sam's error: He did not rebundle 14 ones into 1 ten 4 ones and place the 1 ten in the tens place.

Homework Set 11

1. 7 hundreds, 12 tens and 12 ones.
4. The middle calculation is the hardest because it requires 3 rebundlings, one across 0.
6. (1) No rebundling needed for (a)-(f). (2)-(4): rebundle across 0 in the ones, tens and hundreds place. (5) ones, ones, tens, etc.
7. Practice 2C, Problem 5. This is clearly an addition problem; the teacher's solution consists of the sum calculated using the addition algorithm (no diagram needed).
   Problem 10: use the bar diagram



$$\begin{array}{rr} 2572 & 5000 \\ +\ \ 955 & -\ 3527 \\ \hline 3527 & 1473 \end{array}$$

(a) Mr. Johnson spent \$3527.

(b) He had \$1473 left.

8. (b) Julie rebundled 1 ten as 10 ones, but failed to record that there were only 9 tens left.

Homework Set 12

2. Motivation (reread the first paragraph of Section 2.2 on the uses of word problems). This is reinforced on pages 54-56 and 65-67 of Primary Math 3A, where students use the algorithm to solve similar word problems with larger numbers.

Homework Set 13

1. Include the word "equally," write in complete sentences and ask "How many in each?" and "How many left?"
3. Hint: Read Example 7.2c) on pg. 40 of *Elementary Math for Teachers*, and also the Preface of Primary Math 3A.

Homework Set 14

1. b) Place value.
4. There is a trade-off between accuracy and ease of computation, so this problem has various possible answers.
6. a) For subtraction, round both up (this keeps the difference small).
   c) Be sure to *under*estimate.

Homework Set 15

1. a) Before. First comes the basic place value idea (in thought bubble); once this is understood, it is then applied to every chip in the chip picture.
2. b) Estimating quotients.
   c) The Primary Math books follow a "Concrete ⇒ Pictorial ⇒ Abstract" (see the Preface of Primary Math 5A). Grade 5 division is at the last stage, which involves working with numbers only.
3. Problem 10. For fifth graders, no diagram is needed.
4. Tracy made a place value error; explain what it is.

Homework Set 16

3. Note that algebraic expressions cannot contain an = sign; $y \div 0$ is not an algebraic expression because it does not represent a number for any value of y.
8. a) Start with the sentence: *Let E be the number of stickers that Emily has.*
9. b) 8; to emphasize that *any* letter can be used to signify a number.
   c) The commutative property of multiplication.
10. c) Problem 8.
    Henry's age: x years.
    Betty's age: $3x$ years.
    (a) Peter's age: $3x + 4$ years.
    (b) If $x = 4$, then Peter is $2x + 4 = 12 + 4 = 16$ years old.

Homework Set 17

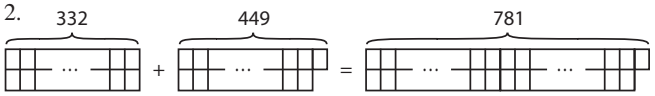
1. h) $2^{14} = 2^{10} \cdot 2^4 = 1024 \cdot 16 =$16,000+24 $\cdot$ 16 =16,384.
2. a) $13 \times 15 = 14^2 - 1 = 196 - 1 = 195$.
   g) $34{,}000 \overset{2}{\div} 50 = 68{,}400 \div 100 = 684$.
   h) $142^2 - 140^2 = (142 + 140)(142 - 140) = 282 \cdot 2 = 564$.
6. a) $14 \times 16 = 15^2 - 1 = 224$, etc.
9. Hint: what is the area of the unshaded region?

Homework Set 18

1. c) $4096 \div 32 = 2^{14} \div 2^5 = 2^9 = 512$.
2. c) $51200\cancel{0} \div 32\cancel{0} = (512 \div 32) \times 100 = (2^9 \div 2^5) \times 100 = 2^4 \times 100 = 3200$.
7. c) Note that $(ac)^0 = 1$.
9. Expand $N^2 = (2380 \times 10^4 + 5723)^2$ using $(a + b)^2 = a^2 + 2ab + b^2$ to get $2380^2 \times 10^8 + 2 \cdot 2380 \cdot 5723 \times 10^4 + 5723^2$. Write down 2380^2 (found by calculator) followed by 8 zeros, etc. Finish by doing a 15-digit column addition.

Homework Set 19

2.



4. b) Given any two odd numbers A and B then, by Definition 1.1, we can write $A = 2k + 1$ and $B = 2\ell + 1$ for some whole numbers k and ℓ. Their sum is ...

Homework Set 20

7. *Proof.* Consider a 4-digit number N with digits $abcd$, so $N = 1000a + 100b + 10c + d$ ...

Homework Set 21

4. b) Given a whole number, N, each factor a has a partner $b = \frac{N}{a}$ unless $a = b$ for some a. *What happens if $a = b$?*

5 c) Start by writing down the prime factorization of 2,400,000.
   d) Again, start by prime factoring 18!.

6. b) Note that 25 contributes two factors of 5.

Homework Set 22

3. a) "Let's look at 9" (or 15, 21, etc.)
   c) Extend the table until you find a counterexample. Keep going; don't lose heart.
4. *Proof.* Given 3 consecutive whole numbers, let x be the smallest. The next two numbers are $x+1$ and $x+2$. Then their sum is . . .
7. No; the next entry q_5 is a counterexample.

Homework Set 23

2. By Definition 5.1, GCF(a,b) is a factor of a. But the largest factor of a is a itself; thus GCF(a,b) $\le a$. On the other hand, if b is a multiple of a, then . . .
7. a) Adding and subtracting fractions with denominators that are different, and one is not a multiple of the other.
9. As Gear 1 turns one revolution, Gear 2 turns $\frac{192}{320}$ revolution. After Gear 1 has made N revolutions, Gear 2 has made $\frac{192N}{320}$ revolution. We wish to find the smallest N such that this fraction is a whole number; by Definition 5.8 this is . . .

Homework Set 24

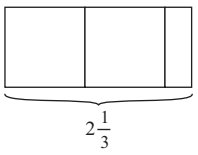
4. b) Equivalent fractions, or expressing fractions in simplest form.
7. To give an algorithm, complete the sentence: for each positive fraction $\frac{a}{b}$, the fraction ____ lies between 0 and $\frac{a}{b}$.

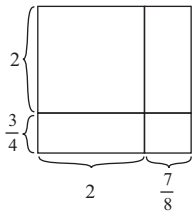
Homework Set 25

4. a) $28\frac{2}{7} - 3\frac{6}{7} = 27\frac{9}{7} - 3\frac{6}{7} = 24\frac{3}{7}$.
7. Hint: your answer should include the formula $46 = (6 \times 7) + 4$.

Homework Set 26

1. b) $25 \times 320 = 100 \times \frac{32}{4}$ tens $= 800$.
3. b) $24\frac{1}{4} \times 1\frac{2}{3} \approx 24 \times 1\frac{2}{3} \approx 24 + (\frac{2}{3}$ of $24) = \cdots$
7. a) Form a rectangle from 6 copies of the lefthand diagram below.
   c) Use the righthand diagram below.





8. Draw a rectangle with side lengths labeled $\frac{2}{3}$ and $\frac{1}{2} + \frac{1}{3}$, subdivide and match the parts with the equation.
9. Draw an area model like the one shown above for HW Problem 7c.
11. c) Hint: This asks: "32 feet is 4 sides of what length?"

Homework Set 27

2. Use the chart on page 146.
3. b) Start by drawing a bar consisting of 9 small squares. Shade two squares and label them "$\frac{2}{3}$ hour."

Homework Set 28

4. Use the chart on page 146. Start by writing down the interpretive question. Then choose a unit, and build your problem.
   a) Interpretive question: 135 is how many units of size 5? Unit: inches (say). Possible problem: A brick wall is 135 inches wide. If each brick is 5 inches wide, how many bricks are in each row of the wall?
6. d) Lisa had enough money to buy 2 shirts or 3 hats. She bought $\cdots$ and had $\frac{1}{6}$ of her money left. How much . . . .
7. See page 89. Begin: Let x be the unknown number. We're told: $\frac{8}{9}x = 512$.

Homework Set 29

5. a) Kyle tried to do too many steps at once, so many that he lost track of what was left not crossed out.
   b)
$$\begin{aligned}\frac{20}{21} \times \frac{27}{5} \div \frac{18}{35} &= \frac{20}{21} \times \frac{27}{5} \times \frac{35}{18} \\ &= \frac{2\cdot 2\cdot 5}{3\cdot 7} \times \frac{3\cdot 3\cdot 3}{5} \times \frac{5\cdot 7}{3\cdot 2\cdot 3} \\ &= \frac{2\cdot \cancel{2}\cdot \cancel{5}\cdot \cancel{3}\cdot \cancel{3}\cdot \cancel{3}\cdot \cancel{5}\cdot 5\cdot \cancel{7}}{\cancel{3}\cdot \cancel{7}\cdot \cancel{5}\cdot \cancel{3}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{3}} \\ &= 10. \qquad \textit{(Each step on a new line.)}\end{aligned}$$
6. g) $\frac{\frac{3}{8}(2+1)}{\frac{3}{8}(2-1)} = 3$. i) $\frac{120}{31} = 3\frac{27}{31}$.

Homework Set 30

6. a) Following the hint, $\frac{a}{b} = \frac{cx}{dx}$. Since $x \neq 0$, what can you do next?
8. From the given information, we can write

   Black: 1 unit
   Red: $\frac{6}{11}$ unit
   White: $\frac{2}{3}$(number of red) = $\frac{2}{3} \cdot \frac{6}{11}$ unit = $\frac{4}{11}$ unit.

   Thus the ratio of white to black marbles is $\frac{4}{11} : 1$, or $4 : 11$.

Homework Set 31

4. Following the hint: Jill's final amount: $(x - 34)$ dollars
   Karen's final amount: $(x - 16)$ dollars.
   Ratio: $(x - 34) : (x - 16)$
   Use this ratio to setup a proportion and solve.

Homework Set 32

2. If the original price of the shirt is x dollars, then the sales price is $(0.65)x$. Why? Use this expression to setup an equation and solve.

Homework Set 33

5. $\dfrac{0.3\ \ell}{\cancel{\text{m}^2}\ \cancel{\text{day}}} \times 6000\ \cancel{\text{m}^2} \times 7\ \cancel{\text{days}} = \_\_\_\_\ \ell$
6. Hint: It's not 50 mph!
7. Mimic the solution to Example 4.6. Answer: 21 hours.

Homework Set 34

4. Hint: How is the symbol "$-$" being used in Definition 1.1? In Rule 1?
5. b) Be careful! There is a subtle distinction between integer addition problems and subtraction problems.

   To construct a problem that unambiguously involves adding negative numbers, think of a situation that involves successive changes, each of which can be positive or negative. For example: A stock market fund went up \$343 on Monday, but went down \$44 on Tuesday. What was the total change for these two days?

   In contrast, the following is a take-away subtraction problem: Brad had \$343 in the bank. He used \$44 of it to buy a shirt. How much does he have now?
6. c) Is this true for $a = 3$? For $a = -3$?

Homework Set 35

1. a) Hint: $a - (-b) = a + (-(-b))$ by Rule 2. Now apply Rule 1.
3. a) $(23 + 26 + (-13)) \cdot 25 = 36 \overset{4}{\cdot} 25 = 9 \cdot 100 = 900$.
6. a) Start by writing: $2^{12} \cdot 12^{-5} \cdot 15^8 = 2^{12} \cdot (3 \cdot 2^2)^{-5} \cdot (3 \cdot 5)^8$.

Homework Set 36

1. Copy the proof of Theorem 3.2 replacing a with 7.
7. Use Rule 3.
9. Hint: Try various positive and negative numbers for a and b.
11. Use Theorem 3.6 to show that $(-1)a > (-1)b$, then use Rule 3 (or HW Problem 7) to show that $-a > -b$, then apply Order Property 3.
13. See the solution above to Problem 11.

Homework Set 37

2. a) $17.32 - 9.97 = 17.32 - 10 + .03 = 7.35$.

   i) Hint: multiply by .001 by shifting the decimal point.
5. d) All of them.
8. Follow Example 4.2 on page 73. Proceed in 3 stages (tens, ones, tenths places), the first being:

$$\begin{array}{r|l} & 1 \qquad\qquad < \text{ 1 ten given to each} \\ 3 & 5\ 6.\ 4 \\ & \underline{-3} \qquad\quad < \text{ total of 3 tens distributed} \\ & 2\ \ 6 \qquad\quad < \text{ 26 ones to distribute} \end{array}$$

10. Shift decimals to put each in the form $x \div 47$ for some x.
11. Start by rewriting as division by a whole number.

Homework Set 38

3. h) Use long division. Keep going until it repeats.
8. d) Multiply each by $\frac{5}{5}$.

Homework Set 39

6. b) Write as $(2.236 \pm .0001)(3.1416 \pm .0001)$.

Homework Set 40

6. Start by supposing that $\sqrt{\sqrt{2}} = \frac{a}{b}$ for whole numbers a and b.